[9] Vibrations which can absorb light waves of definite length involve *quantitative* limits in the field of forces acting upon the electrons. In the absence of quantitative measurements of intramolecular potentials, the quantitative factors are at least indicated by the terms used. The range of wave-lengths for visible color is extraordinarily narrow, as is well known.

[10] The negative charge on the nitrogen atom ${}^{-}_{-}N^{-}$ which holds the oxidized atom ${}^{+}_{+}C^{+}_{+}$ and the reduced atom ${}^{+}_{+}C^{-}_{+}$ close together presumably repels the electrons in ${}^{+}_{+}C^{-}_{+}$ and its force is added to the intra-atomic pull of the positive nucleus of the ${}^{+}_{+}C^{+}_{-}$ atom itself. But it is thought that the effect of ${}^{-}_{-}N^{-}$ is of minor moment because it is an ammono-nitrogen atom as in ammonia with practically *no reducing power* of moment—as little indeed as the reducing power of $O^{=}$ in water—presumably on account of the stability of the nitrogen atom with its full complement of eight electrons found in $N^{\equiv}$. The effect of oxidizing-reduction potentials depends on the *activity* (perhaps the mobility or freedom) of charges according to all the evidence we have.

[11] W. Ostwald, *Zs. phys. Chem.*, **15**, 399 (1894). A description is given in Stieglitz *Qualitative Analysis*, Vol. I, p. 253 (1911) (Century Co.).

[12] The explanation of this effect is given in the author's *Qualitative Analysis*, Vol. I, p. 292 (1911).

[13] Nietzki, *Chemie der Organischen Farbstoffe*, p. 162 (1906).

[14] A combination of hydroquinol against quinone, both in sodium nitrate solutions, showed a chemometer reading of 5; addition of sodium hydroxide to the hydroquinol raised the needle to line 13, and addition of acid to the quinone lifted the needle to line 17. The experiment has only qualitative significance, but clearly supports the views expressed.

[15] Stieglitz and Slimmer, *Amer. Chem. J.*, **36**, 661 (1904) and a forthcoming article by H. V. Tartar and Stieglitz in the *J. Amer. Chem. Soc.*

[16] Certain authors have assumed that there is inter-atomic migration of electrons in color production. Cf. Adams and Rosenstein, *J. Amer. Chem. Soc.*, **36**, 1452 (1914) and H. L. Wells (see the next reference).

[17] H. L. Wells, *Amer. J. Sci.*, **3**, 417 (1922) gives a number of excellent illustrations of this relation. Cf. Willstaetter and Piccard, *Ber. D. chem. Ges.*, **41**, 1465 (1908).

THE ASYMMETRY IN THE DISTRIBUTION OF STELLAR VELOCITIES

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The outstanding features in our knowledge of stellar velocities are the existence of stream-motion, the dependence of the velocities upon absolute magnitude and spectral type, and the asymmetry in the velocity-distribution. The asymmetry in the distribution of velocities was first found by B. Boss[1] in a study of the stars of measured parallaxes and radial velocities. It was independently found by Adams and Joy[2] in studying the space-velocities of 37 stars of large radial velocities. The general character of this phenomenon was established by the present author[3] from a study of the distribution of the three-dimensional velocity-vectors of 1300 stars of spectral types F to M. The same general effect was found by Boss, Raymond, and Wilson[4] from a study of the space-velocities of 520 stars of measured parallaxes.

This asymmetry is best seen among stars of very large velocity and shows itself as a tendency for stars of high speed to move towards one hemisphere of the sky, namely, that limited by galactic longitudes 160° and 340°. Its effect can be traced even among stars of ordinary speed and produces a difference between the most frequent velocity-vector and the mean velocity.[3] It was suggested by Oort[5] that the asymmetry might be due to the existence of two groups of stars of different internal velocities and of different group-motion. This division into two groups is not distinct and the phenomenon seems rather to be a continuous change in the group-motion with increasing internal velocity.

The purpose of this communication is to show that the asymmetry is a general phenomenon among all the objects in the universe for which we have data regarding velocities, and that it may be reconciled with the existence of a fundamental system of reference for the velocity-vectors. This was found by a deductive course of reasoning, although it might just as well have been found inductively.

Suppose for the moment that there exists a fundamental system of reference, such that excessive velocities referred to it are very infrequent. To express this in a mathematical form we may assume that the velocities referred to this system are distributed according to a spherical distribution-law. The probability of the occurrence, of a velocity (ξ, η, ζ) referred to this fundamental system of reference, is thus proportional to

$$F_1 = e^{-k^2(\xi^2 + \eta^2 + \zeta^2)} \tag{1}$$

As a coördinate system we shall use the galactic system, the X-axis towards 0° galactic longitude, and the Z-axis towards the north galactic pole. All the velocities have been corrected for a standard solar velocity of 20 km./sec. towards the point $\alpha = 270°$, $\delta = +30°$. The sun's velocity-components in this system of coördinates[3] are then $x = +17.0$, $y = +7.4$, $z = +7.4$ km. Referred to this standard origin, we can write equation (1)

$$F_1 = e^{-k^2[(x + x_0)^2 + (y + y_0)^2 + (z + z_0)^2]} \tag{2}$$

where x_0, y_0 and z_0 are the velocity components of our origin referred to the fundamental system of reference.

In the absence of this external effect the velocities of all groups of stars are supposed to be distributed symmetrically around a common origin, whose coördinates in our adopted system are α, β and γ. In making this assumption we neglect the difference in group-motion between the central group and the Taurus group,[3,6] which difference for our purpose is of little importance. As a general expression for a symmetrical velocity-distribution we shall use the function

$$F_2 = \frac{1}{\pi^{3/2}} \sum_v N_v l_v m_v n_v e^{-l_v^2(x'-\alpha')^2 - m_v^2(y'-\beta')^2 - n_v^2(z'-\gamma')^2} \qquad (3)$$

This function expresses the sum of a series of concentric and co-axial ellipsoidal distributions. The velocity α', β', γ' is the velocity of the common centroid, referred to the principal axes and to our previously adopted origin. This division into several ellipsoidal distributions is not necessarily a physical one, but may be regarded as a purely statistical representation, although we may regard each one as a group, which is in a certain sense homogeneous.

Referring our velocities throughout to the common principal axes, we shall for the present drop the accents.

If we now suppose that the frequencies of the actual velocities depend simultaneously upon the two probabilities F_1 and F_2, we find the probability of a certain velocity (x, y, z) expressed by the product of both,

$$F dxdydz = e^{-k^2[(x+x_0)^2 + (y+y_0)^2 + (z+z_0)^2]} dxdydz \times \sum_v A_v e^{-l_v^2(x-\alpha)^2 - m_v^2(y-\beta)^2 - n_v^2(z-\gamma)^2} \qquad (4)$$

The function F can be written

$$\left.\begin{array}{lll} F = \sum_v B_v e^{-h_1^2(x-c_1)^2 - h_2^2(y-c_2)^2 - h_3^2(z-c_3)^2} & & \\ h_1^2 = l_v^2 + k^2 & c_1 = -2a_v^2k^2(x_0+\alpha) + \alpha & a_v^2 = \dfrac{1}{2h_1^2} \\ h_2^2 = m_v^2 + k^2 & c_2 = -2b_v^2k^2(y_0+\beta) + \beta & b_v^2 = \dfrac{1}{2h_2^2} \\ h_3^2 = n_v^2 + k^2 & c_3 = -2c_v^2k^2(z_0+\gamma) + \gamma & c_v^2 = \dfrac{1}{2h_3^2} \end{array}\right\} \qquad (5)$$

This function represents a sum of ellipsoidal distribution-functions, whose centers are shifted in a direction nearly opposite to that of the "absolute" translation of our origin, by an amount which increases with the internal motion of the group. For stars of small relative motions, for which l, m and n are large as compared with k, the center of the frequency function approaches the limit

$$x = \alpha, \quad y = \beta, \quad z = \gamma$$

For stars whose relative velocities are extremely large the center of the frequency-function approaches the other limit

$$x = -x_0, \quad y = -y_0, \quad z = -z_0$$

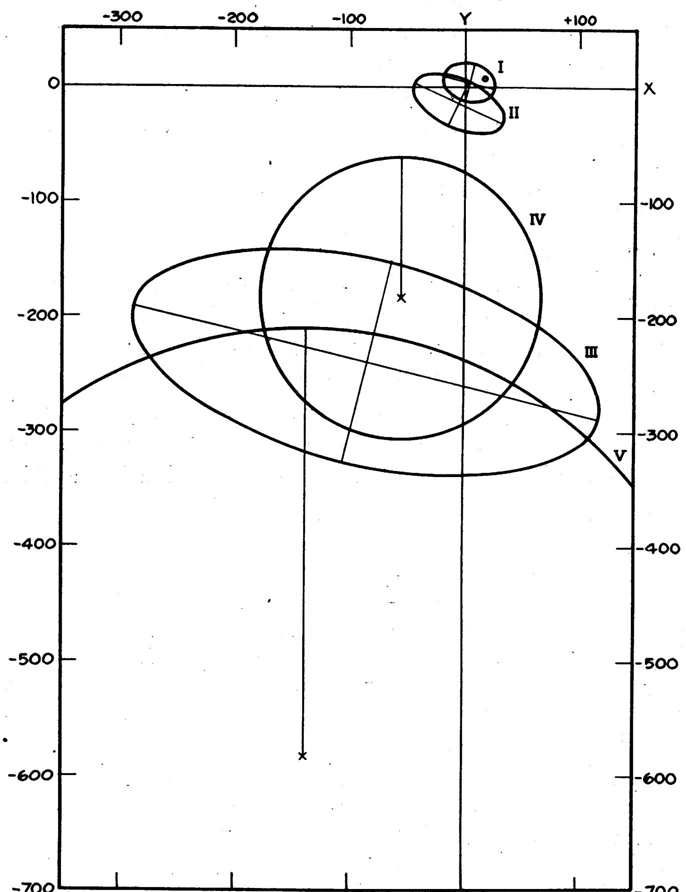
In a general way this displacement of the center of the velocity-distribution with increasing internal motions is borne out strongly by stars, globular clusters and spiral nebulae. The velocity-distribution for stars of spectral type F to M for which space-velocities have been computed from proper motions, radial velocities and spectroscopic parallaxes is represented by the sum of two ellipsoidal frequency-functions and the method derived in a previous study[3,6] was employed. The only difference is that the equations of condition used in these investigations were given additional weights inversely proportional to the square roots of the numbers of velocity-vectors terminating in the different parallelopipeds. Probably there are many more than two homogeneous velocity-groups, and the splitting up in two groups may be entirely artificial, but a representation by the sum of two ellipsoidal frequency-functions represents the data sufficiently well. There exists a group of stars of space-velocities exceeding 150 km./sec., which seems to form a separate group, and they were treated accordingly as group III in the table. Groups I and II are thus stars of moderate speed. Group IV consists of the globular clusters and group V of the spiral nebulae. For the two latter groups the radial velocities alone are available and the distribution is supposed to be spherical. Lundmark has computed for other purposes these two groups from data compiled by him, mainly from Slipher's determination, and has kindly allowed me to use his computations.

The following table gives the results of this analysis.

		No.	c_1	c_2	c_3
I	Stars of moderate velocity	768	+ 3.6	+ 4.4	+ 0.8
II	Stars of moderate velocity	510	− 5.3	− 14.8	− 0.4
III	Stars of high velocity.....	21	− 85	−240	− 10
IV	Globular clusters........	17	− 55	−184	− 39
V	Spiral nebulae..........	42	−137	−583	−220

	a	b	c	L_1	B_1	B_2
I.........	22.66	16.56	10.52	165.4°	−0.3°	+ 2.7°
II........	42.09	22.10	27.84	155.3	+1.1	−12.7
III.......	210	84	89	167	+5	+ 3
IV........	122	122	122			
V........	373	373	373			

The centers of the velocity groups are given in km./sec. referred to the standard origin and to the axes defined in the beginning. The dispersions a, b, and c (mean square velocities) along the principal axes are likewise expressed in km./sec. L_1 and B_1 are the galactic longitudes and latitudes of the major axis a, and B_2 is the galactic latitude of the axis, along which the dispersion is equal to b. In the diagram are plotted the intersections of the velocity-ellipsoids with the galactic plane for the five different groups. The velocity of the sun is denoted by the symbol ⊙. The general tendency for the center of the velocity-distribution to be shifted in the general direction of galactic longitude about 250°, by an amount which increases with the internal velocities of the objects in the group, is clearly shown. According to equation (5) this shift ought to be a linear function of b^2, as the axis b is nearly parallel to the shift itself. Although this is not strictly the case, there can be no doubt about the general tendency. Furthermore the quantity k may not be strictly the same for the different groups, even if x_0, y_0 and z_0 are constant.



Intersections with the galactic plane of the velocity-ellipsoids for different groups of stars: I and II, stars of moderate velocity; III, stars of high velocity; IV, globular clusters; V, spiral nebulae. The diagram shows the progressive change in group-motion with increase in the internal motions of the respective groups. Ordinates and abscissae indicate kilometers per second.

This phenomenon cannot be due to a local gravitational field, as it holds even in the most remote regions accessible to observation. It