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Ethelbert Nwakuche Chukwu

# Economic Dynamics of All Members of the United Nations

*Second Edition*

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# Economic Dynamics of All Members of the United Nations

Second Edition



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# Preface

After the meetings of Asian and European Leaders in Beijing, China on April 12, 2008, and their joint stand on the solution of the world economic crises at the Washington, November 15, 2008, G20 meeting and the subsequent economic summit of World Leaders in London, Chief Financial Leaders from the world's top seven economic powers pledged, as reported by Jeannine Aversa of the Associated Press, Saturday, April 25, 2009, in the News and Observer:

“The group of seven participants Japan, Germany, France, Britain, Italy and Canada promised to provide the necessary fiscal tonic to turn around their troubled economies as follows—tax cuts or increased government spending; commitment to act together to restore jobs and growth.”

The USA Treasury Secretary, Timothy Geithner and his counterparts made this promise in a joint statement. The news report touched upon new financial commitments to raise \$500 billion for IMF lending and the difficulty of getting China, Brazil, Russia, India, and Saudi Arabia on board. A bigger say in the operation of IMF was needed to be put in place.

The creation of jobs and the restoration of economic growth are the main arguments embodied in Chukwu's current book. The path of cooperation is emphasized. Implicitly they embraced and accepted the conclusions of the book, “The Omega Problem of All Members of the United Nations”.

By affirming the joint statement “We will take whatever actions are necessary” to bring that about—to prevent “a crisis of this magnitude from occurring again”, “fixing financial institutions in the U.S. and World wide and jump-starting lending”. What is accepted implicitly is the analysis of Chukwu on Differential Models and Neutral Systems for controlling the Wealth of Nations, World Scientific 2001, Singapore, pp. 293–296 as highlighted in the forward.

# Acknowledgments

I thank Emeka Chukwu, my MATLAB Programming Assistant.

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Dr. C. K. Chui of Stanford University, Editor in Chief of Atlantis is incomparable and great. To the publisher of Atlantis Press, Dr. Keith Jones, and the Founder Zeger Karssen, my gratitude is beyond belief. Thank you all.

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# Introduction

In a recent paper the author postulated that the most important applied mathematics problem since the world began is connected with the worldwide conquest of scarcity subject to the values of love and goodness. (See Genesis 3:17–18; Isaiah 58:6–8; 9–12; Malachi 3:10–12; and Matthew 25:31–40). The production of goods and services and their distribution are considered to be the core of applied mathematics problem in the sense that mathematical economic state of all nations—the gross domestic product, interest rate, employment, value of capital stock, prices (and therefore inflation), and cumulative balance of payment—is mathematically derived and identified as a dynamical system with interacting solidarity matrix. Included in the model are government strategies—generalized taxes, autonomous government outlay, exchange rate, tariff, trade policy or distance between trading nations, money supply and its flows, autonomous net capital outflow to foreigners by government. Included also is the representative private firms strategies—autonomous consumption, autonomous investment, autonomous net export, autonomous money demand, labor productivity, wage rate, autonomous income consumption intercept, and autonomous price intercept. By “autonomous” we mean that these do not depend on variables of the economic state. It is possible to include nongovernment organization (NGO) contributions. The production of abundant goods and services driven by government and private strategies is the fundamental aim. With the mathematical economic model which joins all economies of nations, one asks the question whether it is possible to steer each economic state of a nation from low growth of GDP, high interest rate, low employment, low value of capital stock, high inflation, and little cumulative balance of payment to the state of abundance of goods and services. It is then possible, if humans are willing, to feed the hungry, clothe the naked, shelter the homeless, take care of the sick and heal them, visit and care for prisoners, welcome strangers and foreigners to our home and to our countries. Thus, we uphold the uplifting value of love and goodness. How can this be done? First, we take the underlying Applied Mathematical economic problem of improving the economic state of nations using government and representative firm’s strategies. A measure of economic performance which is to be improved is the gross domestic product GDP. It is a measure of the total income generated through the production of goods and services, the so-called output  $y$ —the supply. Assuming the differential principle of supply and demand and the principle of

rational expectation, we derive the dynamics of the six components of each country's economic state. Using MATLAB computer program and IMF, UN data, all the coefficients of the dynamics are identified. The system is a differential game of pursuit of the form

$$\dot{x}(t) - A_{-1}\dot{x}(t - h) = A_0x(t) + A_1x(t - h) + A_2(x)x(t - h) + B_1p + B_2q,$$

with government strategy  $q$  as the quarry and the representative firms' control function as the pursuer,  $p$ , which can be supplemented by nongovernment organization contribution, NGO. By the method of Hájek [1] and Chukwu [2] the system is converted to a control system with 191 state variables and a control variable  $u$  a member of the Pontryagin difference of the control set of government  $Q$  and the control set of the private firms representative  $P$ .

Once this is set up, one studies the controllability and the optimal growth of the economic state of

$$\dot{x}(t) - A_{-1}\dot{x}(t - h) = A_0x(t) + A_1x(t - h) + A_2(x(t))x(t - h) + Bu,$$

where  $x$  has 191 components corresponding to member states of the UN. Thus,

$$x = [y_1, y_2, y_3, \dots, y_{191}],$$

and  $y_i$  is the GDP of country  $i$ . We aim to have material abundance. For this we need huge resources. These are currently available, e.g., from the Sun. It will take at least 3 billion years to exhaust the energy of the Sun. What is available from the waters of the oceans is tremendous. Included are the tons of sand of the deserts, and of the sea—shores—the sand for the silicon and computer chips, and for building, shelter. Ndu Chukwu in a project at St. Augustines College, Raleigh, North Carolina, a follow-up of NASA Project: "Colonization of the Moon," indicated that even the deserts of Peru, Arizona, California can be watered, colonized, and made to bloom. The down pour of the rainstorm caused by El Niño proved him right. It was documented by CBS which witnessed the fulfillment of the prophecy of Isaiah 35:1-2: "The desert will bloom." There is more arable land for humankind to grow its economy than is recognized. We now present the economic model which joins all economies of all nations and present how it is possible to steer each nation's economic state of low growth of GDP, high interest rate, low employment, low value of capital stock, high inflation, and negative cumulative balance of payment to the state of paradise—high growth rate of GDP, low interest, full employment, low prices (or small inflation), and great cumulative balance of payment. It will be shown that it is possible to steer to this target with minimum investment, and do it in minimum time. All nations of the world can then use this model to implement an optimal economic strategy and usher in universal abundance and prosperity. The needed huge resources are already available in our world. Closely linked with this first section of our study is the issue of longevity. It seems that life can be prolonged beyond our imagination: 500 years and going strong. By caloric restriction and manipulating the anti-aging

genes, and by exercise, humans can live longer. It has been observed that there was a 30 years gain in the age of humans in the last century and that longevity and health are related to the generation of wealth. Countries that have a 5-year advance in longevity compared to other countries have a greater Gross Domestic Product. Increasing health and longevity will enable countries to create greater wealth and prosperity. Citizens will live longer as healthy productive individuals. Thus, GDP increases. The value which was earlier articulated, “Take care of the sick,” now assumes a prominent position as a component of the omega problem of all members of the United Nations.

We briefly outline the contents of the book. In [Chap. 6](#), we use the principle of rational expectations and the principle of differential supply and demand to derive differential mathematical models of the economic state of all members of the United Nations. Later in [Chap. 14](#), these equations are validated with historical data from the International Statistical Yearbook and United Nations National Accounts Statistics: Main Aggregates and Detailed Tables, UN, New York. The generic type of these equations is reproduced for each nation from Chukwu, *Optimal Control of the Growth of Wealth of Nations*, Taylor and Francis 2005 and with Matlab identified in [Chap. 14](#) below, or Chukwu, *Stability and Time-Optimal Control of Hereditary Systems with Application to Economic Dynamics of the US*, 2nd Edition, World Scientific, 2001, Singapore Chapter 1:10. The Economic State consists of the popular ones, six components-Gross Domestic Product, (GDP)  $y$ , Interest Rate  $R$ , Employment (or Unemployment)  $L$ , Value of Capital Stock  $K$ , Prices, (or inflation  $\dot{p}$ ), and cumulative Balance of Payment,  $E$ . The Control Strategies of Government and of the representative private firms are also identified. From these, several real strategies for growth in a nation are highlighted for implementation. The usual analysis for stability, controllability, and permanence are made.

When all the nations are linked up together with an interconnecting function  $f(t) = A_2(x(t))x(t - h)$ , the emerging dynamics is

$$x(t) - A(x(t - h)) = A_0x(t) + A_1 x(t - h) + A_2x(t)x(t - h) + Bu,$$

in which the Matlab programs identify  $A$ ,  $A_0$ ,  $A$ ,  $A_2$  and  $B$ . Here  $u$  is a member of the Pontrygin difference of sets  $P$  and  $Q$ . Here  $x$  has 191 components corresponding to member states of the UN. Thus,

$$x = [y_1, y_2, \dots, y_{191}].$$

$y_i$  is the GDP of country  $i$ .

Cooperation studies can be continued in the direction of the insight of T. G. Hallam, “Community Dynamics in a Homogeneous Environment in Mathematical Ecology,” *Biomathematics*, Vol. 17, p242.

*Biomathematics* ed. T. G. Hallam and S. A. Levin, Springer-Verlag, Berlin-Heidelberg, 1986, and the emergence, if conditions permit, of the so-called “orgy of mutuality.”

In **Chaps. 9** and **1**, we study together the two components of the national economic state, Employment and Gross Domestic when the two nations are interacting and the interacting functions are explicitly defined. The control strategies of the representative firms and the government are explicit and can be used to deduce economic policies. China and USA are two examples when their parameters have been identified. An Appendix of numerical work by E. N. Chukwu and E. Chukwu is displayed.

Detailed studies of the 184 nations are continued in Chap. 12. The book concludes with an overview of studies on longevity and the possibility of eternal life and the resource and life style implications of such an eventuality.

## References

1. Hájek O (1975) Pursuit games. Academic Press, New York
2. Chukwu EN (1996) Universal laws for the control of global economic growth with nonlinear hereditary dynamics. *Appl Math Comput* 78:19–82

# Chapter 1

## Full Employment: Private and Government Policies

Employment is currently decaying in the United States of America. Policies of private firms and government to arrest the situation are now hotly debated. This paper is a readable reflection and extraction from Chukwu's books, *Stability and Time-Optimal Control of Hereditary Systems with Application to the Economic Dynamics of the USA*, 2nd Edition, World Scientific 2001 [1], and "The Omega Problem of all Members of the United Nations", *Atlantis Studies in Mathematics for Engineering and Science*, Volume 6. Series editor, C. K. Chui, Stanford University, U.S.A. and World Scientific [2]. Reasonable and verifiable economic policies are deduced from Chukwu.

The dynamics of employment is given by Eq. (1.10.67) of [1]

$$\dot{L} + \ell_5 p(t) + \sigma_3(t),$$

with the economic state

$$x = [y, R, L, k, p, E].$$

Here

- y is the Gross Domestic Product,
- R is the interest rate,
- L is employment,
- k is the value of capital stock,
- p donates prices,
- E is the cumulative balance of payment,
- $\tau$  is tariff,
- e exchange rate,
- T taxes,
- n is labor productivity of the firms.

---

In this section we highlight the analysis of employment dynamics of the USA.

The profit function is

$$F = y - wL - rk,$$

where  $w$  is the wage of  $L$ , labor per unit time,  $r$  the rent of the use of Capital  $k$ . For the maximization of profit  $F$  we define

$$m(w) = \left[ (1 - \alpha) \frac{1}{w} \right]^{1/\alpha}.$$

Using IMF data and MATLAB program US2.m the following output emerges.

$$\begin{aligned} L_0 &= a_0 = 1.554 \\ \ell_1 &= a_1 = -0.1870, \\ \ell_2 &= m(w)a_2 = 7.7325e^{-(004)} \cdot (01870), \\ \ell_4 &= m(w)a_4 = 7.8295 \cdot (209668), \\ \ell_5 &= m(w)a_5 = 7.8295 \cdot (-161365), \\ \sigma_3 &= m(w)\sigma_4 = 7.7325 \cdot e^{-004} \cdot \sigma_4, \\ \sigma_4 &= x_0 + y_0 + I_0, \end{aligned}$$

$x_0$  firms private net export,  $y_0$  firms autonomous consumption and  $I_0$  autonomous investment

$$q_3 = m(w)q_4 = 7.7325e^{-004} q_4,$$

government control strategy,

$$\begin{aligned} q_4 &= g_0 + z_{s13}M - z_{s14}T(t) + z_{s15}e(t) + z_{s16}\tau(t) + z_{s17}d(t) \\ &= 675.2828 + 0.3447M - 5.490e^{-009}T(t) + 0.0956e(t) + 1.0456e^{-005}\tau(t) \\ z_{s13}M &= 4.787e^{-007}M \\ g_0 &= 675.2828 \end{aligned}$$

Economic Policy for the Growth of Employment,  $L(t)$ .

We observe that

$$\begin{aligned} \sigma_3 &= 7.7325 \cdot e^{-004} (x_0 + y_0 + I_0) \\ &= 7.7325 \cdot e^{-004} (-891936) \end{aligned}$$

is the representative private firms strategy. Increase of autonomous net export; autonomous consumption and autonomous investment will make employment bigger. If the private sector was unable to do this, “government has to step in” by increasing  $g_0$ . For example with  $m(w)$   $g_0$  positive, innovative ways for domestic

food production can be explored decreasing import, promoting export. Local consumption  $y_0$  and investment  $I_0$  can also be pursued, by granting loans to small businesses. President Obama is currently touring Durham, NC to encourage a plant of Cree, Inc., a maker of energy efficient LED Lighting for domestic use and export. The company hopefully with Government can initiate policies that will help the company to produce more cheaply for domestic consumption and export, making net export bigger. An economic stimulus package may be a way to go. In general government strategy can be applied.

$$\begin{aligned} q_3 &= m(w)q_4 \\ &= 7.7325e^{-004} [675.2828 + 0.3447M - 5.490e^{-009}T(t) \\ &\quad + 0.0956e(t) + 1.045^{-005}\tau(t)] \end{aligned}$$

First

$$m(w) = \left[ (1 - \alpha) \frac{1}{w} \right]^{1/\alpha}$$

and be increased by decreasing wages domestically or by outsourcing plants to countries or regions where wages are small and repatriating the huge profit made by smaller taxes on their products by foreign countries. For example, Cree received 39 million dollars in federal stimulus money, opened a plant in China in 2010. Since  $-5.490e^{-009}\bar{T}(t)$  is negative, government can decrease taxes and accelerate the growth of employment  $L(t)$ . This is consistent with increasing the taxes of the very upper class and decreasing those of the middle class and small firms with average decrease of  $T(t)$ .

It may be possible to better manage exchange rate or tariff to incite the bigger growth rate of employment.

A positive trading policy  $d$  may help. In our initial analysis we set  $d = 0$ . All these recommendations are effective when subject to the condition that the private firms control set  $P$  dominates government's  $Q$ :  $Q \subseteq p$ . Details of more complicated situation with other nations' impact on the USA can be found in the latest book by Chukwu. "The Omega Problem of all Members of the United Nations," Atlantis Press World Scientific, 2010. The symbol  $g_0$  can be part of government economic stimulus strategy. We remark that the debate that government should not be too big has been verified. Chukwu's research on the Chinese economy showed how effective the reduction in China of Government with 95 % G.D.P. to 49 %, see E. N. Chukwu, "Modeling and Optimal Control of the Growth of Wealth of Nations with Austria, Australia, and Chinese Examples", 1999. IFAC 14th Triennial World Congress, Beijing, P. R. China, M-5-e-0 2-0657, Italy 5-9, 1999. Government control set of the USA is currently not too big only 22 % of the GDP. Hope is promised for the acceleration of employment growth.



## The IMF's Switch in Time

Joseph E. Stiglitz

Project Syndicate, 5 May 2011

This report is due to Professor Stiglitz.

NEW YORK—The annual spring meeting of the International Monetary Fund was notable in marking the Fund's effort to distance itself from its own long-standing tenets on capital controls and labor-market flexibility. It appears that a new IMF has gradually, and cautiously, emerged under the leadership of Dominique Strauss-Kahn.

Slightly more than 13 years earlier, at the IMF's Hong Kong meeting in 1997, the Fund had attempted to amend its charter in order to gain more leeway to push countries towards capital-market liberalization. The timing could not have been worse: the East Asia crisis was just brewing—a crisis that was largely the result of capital-market liberalization in a region that, given its high savings rate, had no need for it.

That push had been advocated by Western financial markets—and the Western finance ministries that serve them so loyally. Financial deregulation in the United States was a prime cause of the global crisis that erupted in 2008, and financial and capital-market liberalization elsewhere helped spread that “made in the USA” trauma around the world.

The crisis showed that free and unfettered markets are neither efficient nor stable. They also did not necessarily do a good job at setting prices (witness and real-estate bubble), including exchange rates (which are merely the price of one currency in terms of another).

Iceland showed that responding to the crisis by imposing capital controls could help small countries manage its impact. And the US Federal reserve's “quantitative easing” (QEII) made the demise of the ideology of unfettered markets inevitable: money goes to where markets *think* returns are highest. With emerging markets booming, and America and Europe in the doldrums, it was clear that much of the new liquidity being created would find its way to emerging markets. This was especially rue given that America's credit pipeline remained clogged, with many community and regional banks still in a precarious position.

The resulting surge of money into emerging markets has meant that even finance ministers and central-bank governors who are ideologically opposed to intervening believe that they have no choice but to do so. Indeed, country after country has now chosen to intervene in one way or another to prevent their currencies from skyrocketing in value.

Now the IMF has blessed such interventions—but, as a sop to those who are still not convinced, it suggests that they should be used only as a last resort. On the contrary, we should have learned from the crisis that financial markets *need* regulation, and that cross-border capital flows are particularly dangerous. Such regulations should be a key part of any system to ensure financial stability; resorting to them *only* as a last resort is a recipe for continued instability.

There is a wide range of available capital-account management tools, and it is best if countries use a portfolio of them. Even if they are not fully effective, they are typically far better than nothing.

But an even more important change is the link that the IMF has finally drawn between inequality and instability. This crisis was largely a result of America's effort to bolster an economy weakened by vastly increased inequality, through low interest rates and lax regulation (both of which resulted in many people borrowing far beyond their means). The consequences of this excessive indebtedness will take years to undo. But, as another IMF study reminds us, this is not a new pattern.

The crisis has also put to the test long-standing dogmas that blame labor-market rigidity for unemployment, because countries with more flexible wages, like the US, have fared worse. Consumption will remain restrained, while strong and sustainable recovery cannot be based on another debt-fueled bubble.

As unequal as America was before the great recession, the crisis, and the way it has been managed, has led to even greater income inequality, making a recovery all the more difficult. America is setting itself up for its own version of a Japanese-style malaise.

But there are ways out of this dilemma: strengthening collective bargaining, restructuring mortgages, using carrots and sticks to get banks to resume lending, restructuring tax and spending policies to stimulate the economy now through long-term investments, and implementing social policies that ensure opportunity for all. As it is, with almost one-quarter of all income and 40 % of US wealth going to the top 1 % of income earners, America is now less a "land of opportunity" than even "old" Europe.

For progressives, these abysmal facts are part of the standard litany of frustration and justified outrage. What is new is that the IMF has joined the chorus. As Strauss-Kahn concluded in his speech to the Brookings Institution shortly before the Fund's recent meeting: "Ultimately, employment and equity are building blocks of economic stability and prosperity, of political stability and peace. This goes to the heart of the IMF's mandate. It must be placed at the heart of the policy agenda."

Strauss-Kahn is proving himself a sagacious leader of the IMF. We can only hope that governments and financial markets heed his words. Professor Stiglitz rests his case in his usual wisdom.

## Chapter 2

# A Mathematical Solution to Boost the Positive Value of the Cumulative Balance of Payment of the USA

Very recently the President of the USA, set up a bipartisan commission to generate strategies to boost the positive value of the Balance of Payment of the USA. A mathematical solution can be deduced from the following research monographs of the author, E. N. Chukwu.

1. E. N. Chukwu, *Stability and Time Optimal Control of Hereditary Systems with Application to the Economic Dynamics of the US*, 2nd edition, World Scientific, 2001, ISBN 981-02-4674-9. Singapore, New Jersey, London.
2. E. N. Chukwu, *Differential Models and Neutral Systems for Controlling the Wealth of Nations*, World Scientific, 2001, ISBN 9810243812, Singapore, New Jersey, London.
3. *Optimal Control of the Growth of Wealth of Nations*, Taylor and Francis, 2003, ISBN 0-415-26966-0, London, New York.
4. E. N. Chukwu, *A Mathematical Treatment of Economic Cooperation and Competition Among Nations with Nigeria, USA, UK, China and Middle East Examples*, *Mathematics in Science and Engineering*, v 203, Elsevier, 2005, ISBN-13978-0-444-51859-0, Amsterdam, The Netherlands.
5. E. N. Chukwu, *The Omega Problem of all members of the United Nations*, *Atlantis Studies in Mathematics for Engineering and Science*, v 7, World Scientific, pub. Date scheduled Summer 2010.
6. E. N. Chukwu, *Stability and Time Optimal Control of Hereditary Systems*, Academic Press, Boston, 1992.

In these publications, the author invokes the economic “Differential demand and supply principle”, and the “Rational expectations hypothesis” which assumes that the expected values of economic variables are functions of the current and past values. With these assumptions the author derives this hereditary model of cumulative balance of payment,

$$\begin{aligned}
& \dot{E}(t) - b_{17}\dot{E}(t-h) - b_8\dot{R}(t-h) - b_{12}\dot{L}(t-h) - b_4\dot{y}(t-h) \\
& = b_0 + b_7e(t) + b_{15}d(t) + b_{13}\tau(t) + b_1y(t) + b_5R(t) + b_9L(t) + b_3p(t) \quad (2.1) \\
& \quad + b_2y(t-h) + b_6R(t-h) + b_{10}L(t-h) + q_6(t) - r_6(t).
\end{aligned}$$

or

$$\begin{aligned}
& \dot{E}(t) - b_{17}\dot{E}(t-h) - b_8\dot{R}(t-h) - b_{12}\dot{L}(t-h) - b_4\dot{y}(t-h) \\
& = b_1y(t) + b_5R(t) + b_9L(t) + b_3p(t) + b_2y(t-h) + b_6R(t-h) \quad (2.2) \\
& \quad + b_{10}L(t-h) - r_6(t) + q_6(t),
\end{aligned}$$

where

$$-r_6(t) = X_0 \text{ (autonomous net export)} \quad (2.3)$$

$$q_6(t) = b_7e(t) + b_8\tau(t) + b_{15}d(t) - f_0 \quad (2.4)$$

where

$$X_0 = x_0 - M_0 \text{ (autonomous net export), } b_0 = X_0 - f_0, \quad (2.5)$$

$$-r_6(t) = x_0 - M_0 = X_0, \text{ (autonomous net export)} \quad (2.6)$$

$$q_6(t) = b_7e(t) + b_8\tau(t) + b_{15}d(t) - f_0.$$

Here  $f_0$  denotes preferential arrangement (which reduce trade barriers and enhance trade flows between nations)

$X_0$  = autonomous net export,

$q_6(t)$  = government control instruments: exchange rate,  $e$ , tariffs,  $\tau$ , foreign credit interest equalization tax,  $f_0$ , preferential arrangement (which reduce trade barriers and hence trade flows between nations,  $d$ , transportation and distance between partners.).

Indeed if

$x$  =  $[y, R, L, k, p, E]^T$ , (Economic state),

$A_{-1}$  is  $6 \times 6$  matrix,

$A_0$  an  $6 \times 6$  matrix,  $A_1 = 6 \times 6$  matrix,

$B_1$  an  $6 \times 8$  matrix,

$B_2$  an  $6 \times 9$  matrix,

as recorded in [1, Sect. 1.10], and if